Using The Helix Mixing Approach On Floating Offshore Wind Turbines

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Abstract. In recent years dynamic induction control has shown great potential in reducing wake-to-turbine interaction by increasing the mixing in the wake. With these wake mixing methods the thrust force will vary in time. If applied to a floating offshore wind turbine, it will cause the platform to move. In this paper the effect of the Helix mixing approach on a DTU10MW turbine on the TripleSpar platform and its wake is evaluated. When the Helix mixing approach is applied at Strouhal equal to 0.25, the yaw movement is excited close to the eigenfrequency of the platform resulting in significant yaw angles for small blade pitch angles. To understand the impact of the motion on the wake, the yaw motion is simulated using the free wake vortex method as implemented in Qblade. Under laminar inflow, results show that the windspeed at a distance of 5 rotor diameters downstream can be increased by up to 10% compared to a fixed-bottom turbine.

1. Introduction

Offshore wind energy is often regarded as one of the key technologies for the transition to green energy [1]. As of 2021, 13.5% of the total installed wind power in Europe was produced by offshore wind by wind farms located in shallow (shallower than 50 m) waters [2, 3]. However, it is in deeper waters (i.e. deeper than 50 metres) that over 80% of the total wind energy resources for Europe can be found [4]. At these water depths, bottom-fixed technologies become infeasible. Therefore, floating offshore wind turbines (FOWT) will play a key role in enabling access to these energy resources. It is expected that as the technology matures, wind farms will grow in size similar to the offshore fixed-bottom wind turbine farms that already exist.

When turbines are placed in a farm, the wake of an upstream turbine will interact with other turbines in the farm. Research on this interaction has received widespread attention for several decades[5, 6], and it remains a relevant topic to this day [7, 8]. Typically, the goal is to reduce this interaction for the purpose of maximizing power and/or load minimization. One technique that has gained traction over the years is wake redirection [9]. Here the rotors are statically yawed to divert the wake away from downstream turbines. While downwind wind turbines may see an increase in their power production, this goes at the cost of the output of the yawed turbine. Although this method has potential, as wind farms grow in size, it becomes increasingly difficult to deflect the wake in such a way that none of the other turbines in the farm are affected. Furthermore, the yawed turbine will experience increased loading, lowering the lifetime of the turbine [10].

Wake-to-turbine interaction has also received attention for floating wind farms, see for example [11–14]. However, as floating turbines are capable of moving over their 6 degrees of freedom several different approaches have also been explored in literature exploiting this possibility. For example, in [14] the platform is pitched using ballast in the buoys. As the platform is pitched, the wake is either deflected upwards or downwards, depending on the pitch angle. Deflecting the wake downwards towards the sea increases the overall power production of the two-turbine wind farm. In [11] and [13] the floating turbines are misaligned for wake redirection and the resulting platform motion is used to reposition the turbines in the wind farm to further reduce wake overlap between turbines. In [13] a several wind farms of different sizes are optimized, where each wind farm has grid layout with a 7D spacing between turbines. The increase in farm efficiency could be increased by actively optimizing the lay-out, where the actual percentage of gain depends heavily on wind farm size and wind direction.

All the aforementioned techniques are steady state, optimal, solutions. Recently, dynamic approaches have also received attention for mitigating wake effects between turbines. One such approach is the Helix, where each of the blades is individually pitched in a sinusoidal manner. The pitch of each of the blades is out of phase resulting in a non-uniform loading of the turbine [15]. As a result, the Helix applies a varying tilt and yaw moment on the floating turbine. When the floating turbine is subjected to these moments it will start to pitch, roll and yaw. The type of motion is primarily dependent on the type of platform.

This paper aims to explore the two following questions: (1), what will the influence of a dynamic mixing approach, such as the Helix, be on the motions of the platform and, (2), what impact will this motion have on the wake. In this paper the effect of the platform motion on the wake will be explored for the DTU 10MW turbine [16] on a TripleSpar platform [17]. Specifically, it will be explored if this natural motion can be beneficial to further mix the downstream wake and increase power generation for a wind farm. For this analysis two turbines are placed in-line and the wind speed perceived by the second turbine is used to compare different scenario's. The remainder of this paper is organized as follows. Section 2 covers the Helix and the platform motions. Section 3 covers the simulation set-up. Sections 4 and 5 present the result and a short discussion respectively. Section 6 forms the conclusion.

2. The Helix Mixing Approach on a Floating Platform

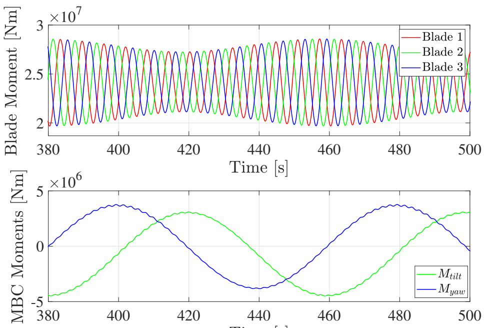
2.1. The Helix

The Helix is applied through setting a sinuisodial signal on the tilt (θ_{tilt}) and yaw (θ_{yaw}) angles. The frequency with which these are varied, are characterized by the non-dimensional Strouhal number St:

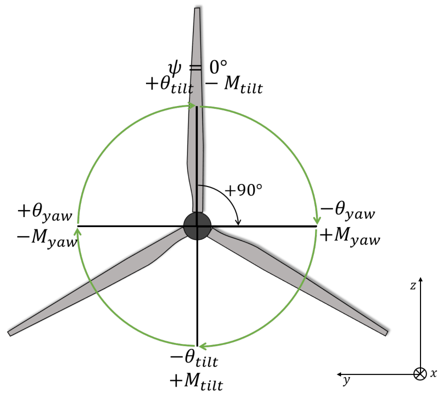
$$St = \frac{f_e D}{V_\infty}, \tag{1}$$

with f_e the pitching frequency [Hz], D the rotor diameter [m] and V_∞ the undisturbed inflow velocity [m/s]. Research on dynamic induction control (DIC) has shown that with low frequency collective pitch [18] significant wake mixing occurs. In [18], an optimal Strouhal number of $St = 0.25$ was found. For the DTU 10MW turbine, with inflow velocity of $V_\infty = 9$ [m/s], one period of either θ_{tilt} or θ_{yaw} takes around 100 seconds while one revolution takes around 9 seconds. These tilt and yaw angles are defined in the non-rotating frame and can be transformed to the rotating frame, and individual blade pitch angles, using the multi-blade coordinate (MBC) transformation [19]:

$$\begin{bmatrix}\theta_1(t)\\ \theta_2(t)\\ \theta_3(t)\end{bmatrix} = \begin{bmatrix}1 & \cos(\psi_1) & \sin(\psi_1)\\ 1 & \cos(\psi_2) & \sin(\psi_2)\\ 1 & \cos(\psi_3) & \sin(\psi_3)\end{bmatrix} \begin{bmatrix}\theta_0(t)\\ \theta_{tilt}(t)\\ \theta_{yaw}(t)\end{bmatrix} = T^{-1}(\psi) \begin{bmatrix}\theta_0(t)\\ \theta_{tilt}(t)\\ \theta_{yaw}(t)\end{bmatrix}. \tag{2}$$



(a) An example of the MBC Transformation



(b) Schematic depiction of location of thrust vector.

Figure 1: Figure 1a: An example of the MBC Transformation for the Helix on a fixed-bottom turbine. Figure 1b: Location of the thrust vector when going through 1 cycle of the Helix. At each of the maxima (+) and minima (-) for the yaw and tilt moment the corresponding pitch angles are denoted.

In equation (2), ψ_i is the azimuth angle of blade i. Angle θ_0 is the collective pitch angle. Likewise, from the out-of-plane bending moments of the blades M_i, it is possible to calculate the yaw and tilt moment applied on the turbine:

$$\begin{bmatrix} M_0(t) \\ M_{tilt}(t) \\ M_{yaw}(t) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ \cos(\psi_1) & \cos(\psi_2) & \cos(\psi_3) \\ \sin(\psi_1) & \sin(\psi_2) & \sin(\psi_3) \end{bmatrix} \begin{bmatrix} M_1(t) \\ M_2(t) \\ M_3(t) \end{bmatrix} = T(\psi) \begin{bmatrix} M_1(t) \\ M_2(t) \\ M_3(t) \end{bmatrix}. \tag{3}$$

An example of this transformation is given in Figure 1a. The top figure shows the blade individual moments, while the bottom figure illustrates the corresponding tilt and yaw moments. The collective moment, M_0, remains constant. Note that even though the yaw and tilt moment/pitch are of low frequency, the individual blades will pitch with roughly the 1P frequency. At any given time each of the turbine blades is at a different pitch angle. The contribution to the total thrust force of each blade is not equally divided among each of the blades. As a result, the overall thrust vector is located off-center of the rotor plane. Figure 1b shows a schematic depiction of the path the resulting thrust vector over the rotor disk through a single cycle of the Helix. The view presented here is a front view of the turbine, with the $x-$axis pointing into the paper. At every 90 degree step in azimuth angle, ψ, the corresponding tilt and yaw moments are denoted. The fixed-frame pitch angles from which these moments arise, are also shown. As the thrust vector moves over the rotor disk it will cause the platform to move. This movement will be analyzed in greater detail in the next section.

2.2. The Helix and Platform Motions

To better understand the motions of an FOWT when applying the Helix mixing approach, several simulations have been carried out in OpenFast [20]. The simulation is open-loop, i.e., for each simulation the Helix is applied and the platform is allowed to freely move. Once steady state has been reached, the gain and phase difference can be deduced from the signals. The inflow velocity is set to be uniform and equal to $V_\infty = 9$ [m/s]. The sea state is assumed to be perfectly smooth, i.e., there are no waves present that could cause further platform movement. For every simulation, the translational and rotational motions of the platform are logged for an extended period of time after steady state operation has been reached. The results are shown in Figure 2 as a Bode Diagram. The red dashed line indicates $St = 0.25$, which is hinted in [18] to be the ideal mixing frequency for the pulse for a second turbine located at a distance of $5D$ downstream. Although it is not yet known if this frequency is also ideal for the Helix, for now $St = 0.25$ is also presumed ideal. The frequency response functions in the top left figure of

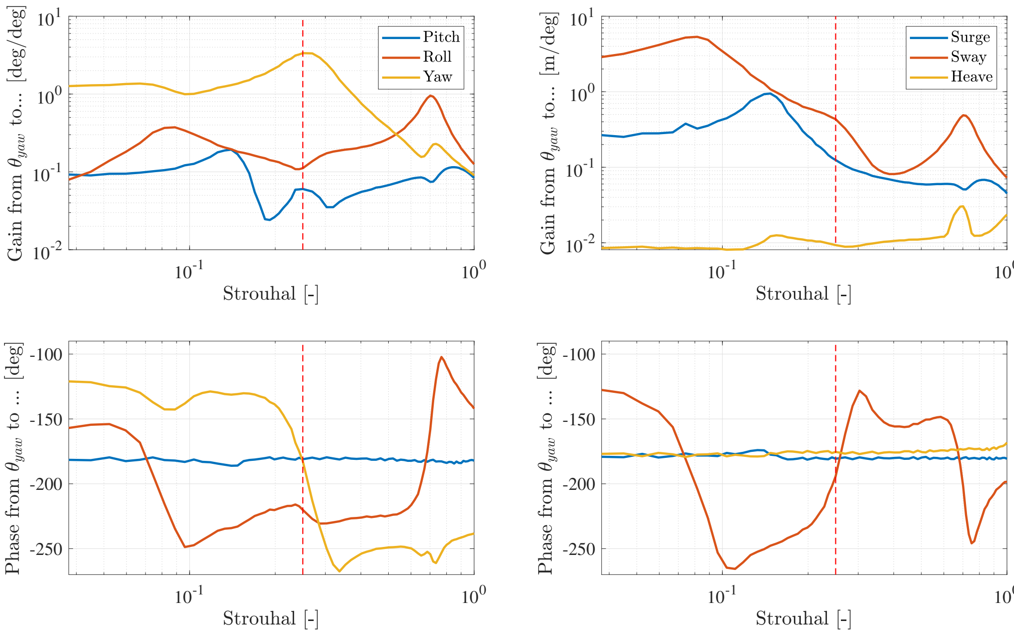


Figure 2: Frequency response functions from θ_{yaw} to platform rotation/translation. The top row shows the relation between blade pitch angles and motions in terms of absolute gain. That is: for every degree of blade pitch angle the FOWT will exhibit the motions with this amplitude The bottom row shows the phase difference for these respectively. For example, if the blade pitch signal is a sine wave with a frequency of $St = 0.25$, then the yaw motion will exhibit a sine like motion with a phase delay of 180°.

Figure 2 show the relation between a degree of blade pitch to degrees of rotation (around any of the axes). The results show that the yaw motion will be the most dominant for an FOWT excited by the Helix. Close to $St = 0.25$ the resonance frequency of the yaw motion can be identified. A typical blade pitch amplitude of $\pm 2°$ at $St = 0.25$ will result in approximately $\pm 6°$ yaw motion. The other rotational motions are also present, but order(s) of magnitude lower. This can primarily be explained by the fact that the roll and pitch motion are naturally dampened. Although the gains for the translational movement are of the same order, the scale of the FOWT plays an important role when interpreting these results. The same input of $\pm 2°$ results in only a swaying movement of $\approx \pm 20$ cm, negligible in comparison to the size of the turbine.

The gain determines the amplitude of the yawing motion. The phase difference between the chosen input and yaw motion will also influence the behaviour of the wake. As the thrust vector moves over the rotor disk, the wind will locally be slowed down more. If, for example, a turbine is kept at a constant positive yaw pitch, the corresponding thrust vector is located to the left, see Figure 1b. This significantly slows down the wake on the left side of the turbine compared to the right side. This effect is shown in Figure 3a, for a fixed-bottom DTU10MW turbine. As a result, the wake starts to deflect towards the left, when looking at the front of the turbine. When the same experiment is done on a floating turbine, the platform will yaw. This yaw angle will cause an additional wake deflection. Depending on the phase-offset of the platform yaw with respect to θ_{yaw}, the wake redirection from the platform yaw might counteract the wake deflection from the yaw pitch. In steady state, and at $St = 0.25$, the phase offset between θ_{yaw} and platform yaw is $-180°$, see Figure 2, which implies that for $\theta_{yaw} = 4°$, the platform yaw angle will be negative. An additional case, where the phase-offset is $-360°$, is also simulated. Both cases are investigated separately. Figure 3b shows the case where the phase is $-180°$, Figure 3c shows the case where there is a $-360°$ phase drop. Figures 3d and 3e show the same yaw angles with $\theta_{yaw} = 0°$ representing a wake redirection case. When the platform is in anti-phase the wake deflection increases, whereas if the platform is in phase wake deflection decreases. These steady state cases show that phase has to be taken into account when the platform is moving. It could be that, even if the platform is moving in phase with θ_{yaw}, the overall wind speed downstream could be higher than a fixed-bottom turbine with the same Helix applied. The exact (dis)advantage of the platform motion will be investigated in a number of different cases introduced in the following section.

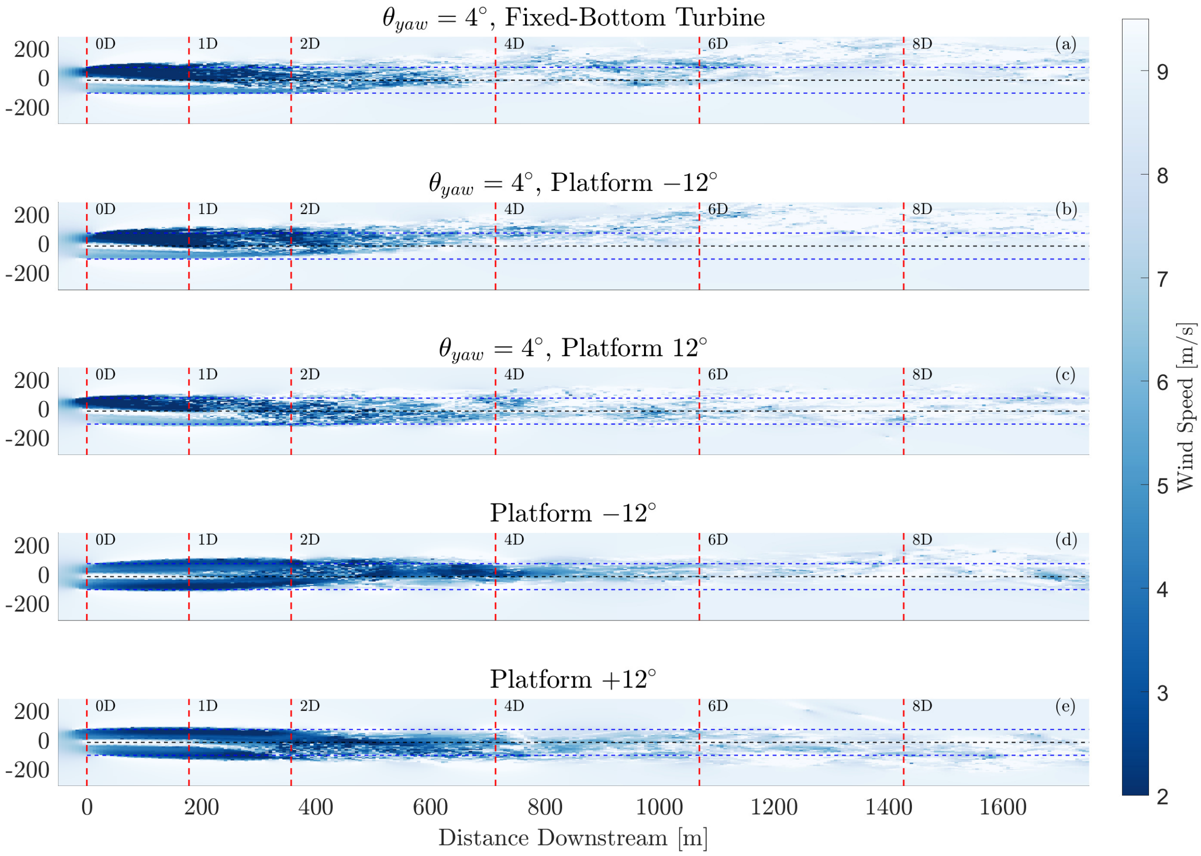


Figure 3: The five different cases showing the effect on the wake of having a static yaw pitch angle as well as how the wake is affected by yaw pitch and platform yaw.

3. Simulation Set-Up

To the knowledge of the authors there currently is no open-source aerodynamic solver capable of explicitly modelling the far wake with an integrated hydrodynamic module. One possibility is to use the results obtained in OpenFast and prescribe a motion to the turbine based on the transfer functions. This is possible in Qblade [21], which is an aero-elastic solver based on the free-wake vortex method. It is developed by TU Berlin. In Qblade, a predefined signal can be applied to move and rotate the turbine at its base, as if it were attached to a floating platform. As the inputs to the FOWT are known, an approximation for the platform motion can be derived using the relations in Figure 2. The exact movement will always be dependent on environmental circumstances. In this paper only the yaw motion will be considered as it is the dominant motion for the system. For comparison, at the excitation frequency the roll motion is approximately 30 times smaller than the yaw motion. A further benefit of singling out the yaw motion is that any difference between bottom-fixed and floating turbines is easier to attribute to the platform yaw motion. On that basis the following five cases are considered for comparison:

(i) Bottom-fixed with constant blade pitch angle (Baseline).
(ii) Helix with 2° pitching amplitude, bottom-fixed.
(iii) Helix with 2° pitching amplitude, 6° yawing motion with -180° w.r.t. θ_{yaw}.
(iv) Helix with 4° pitching amplitude, bottom-fixed.
(v) Helix with 4° pitching amplitude, 12° yawing motion with -180° w.r.t. θ_{yaw}.

For all cases the uniform inflow is set at $V_\infty = 9$ [m/s]. The effect on the wake is evaluated by considering the average wind velocity perceived by a hypothetical DTU10MW wind turbine positioned at 5 rotor diameters downwind. The phase difference of the yawing motion with respect to the θ_{yaw} is kept at -180°. Based on the discussion in Section 2.2 it is important to consider a range of phase differences. Another four cases are considered for this, each with a different phase difference with respect to θ_{yaw}. Case (ii) serves as reference for these cases:

(vi) Helix with 2° pitching amplitude, 6° yawing motion with 0° w.r.t. θ_{yaw}.
(vii) Helix with 2° pitching amplitude, 6° yawing motion with -90° w.r.t. θ_{yaw}.
(viii) Helix with 2° pitching amplitude, 6° yawing motion with -180° w.r.t. θ_{yaw}.
(ix) Helix with 2° pitching amplitude, 6° yawing motion with -270° w.r.t. θ_{yaw}.

Like with the previous cases, the inflow is uniform and set at $V_\infty = 9$ [m/s]. As with the previous cases, the effect on the wake is evaluated by considering the average wind velocity perceived by a hypothetical DTU10MW wind turbine positioned at 5 rotor diameters downwind. For the simulation the ground effect is not taken into account. The free-wake vortex settings are summarized in Table 1. The simulated time is 800 seconds, of which the first 200 are

Wake Variables	**Setting**	**Vortex Variables**	**Setting**
Wake Relaxation	1 (No relaxation)	Initial Core Radius in	0.05% Chord
Max. Wake Elements	200000 [-]	Vortex Viscosity	1100 [-]
Max. Wake Distance	100 Rotor Diameters	Vortex Strain	Off
Wake Reduction Factor	0.001 [-]	Trailing Vortices	Enabled
Near Wake Length	0.5 Revolutions	Shed Vortices	Enabled
Wake Zone 1/2/3 Length	6/12/20 Revolutions		
Time Step	0.1 [s]		
Azimuthal Step	$\Delta\psi = 5.3^\circ$ [deg]		

Table 1: Numerical simulation settings used in all Qblade simulations.

disregarded as the wake initializes to steady state. The time step is set at 0.1 seconds, which results in an azimuthal step of $\Delta\psi = 5.3°$. This step size was found to be a good trade-off between computational time and accuracy [22]. The number of wake elements and the normalized wake distance ensure that the wake travels a distance that is far enough for analysis. Wake zones 1, 2 and 3 are different zones with a length defined by rotational velocity of the turbine of varying fidelity. Between each zone a number of points is removed from the wake to decrease computation time. This process is explained in detail in [23]. The near wake zone, wake reduction factor and all vortex variables are all left unchanged from the reference values. Values for initial core radius and vortex viscosity are based on the research presented in [24].

4. Results

In this section, the results for the cases described in Section 3 are presented. First, cases (i)-(v) are considered, where the Helix is compared to a baseline constant pitch angle case and presents the comparison between floating and fixed bottom turbines. Figure 4 shows the average wind velocity at 5D downwind over time. Despite the irregular behaviour of each of the velocity profiles, a distinct difference between the cases can be spotted. In line with previous research [15], the wake mixing introduced by the Helix significantly increases the wind speed downstream. A further notable result can be recognized by comparing cases (iii) and (iv). The additional yaw motion from the platform significantly raises the average wind speed downstream. A similar result is seen when comparing case (iv) with case (v). On average the green line, corresponding to the floating turbine, has the highest wind velocity. The results are also summarized in Table

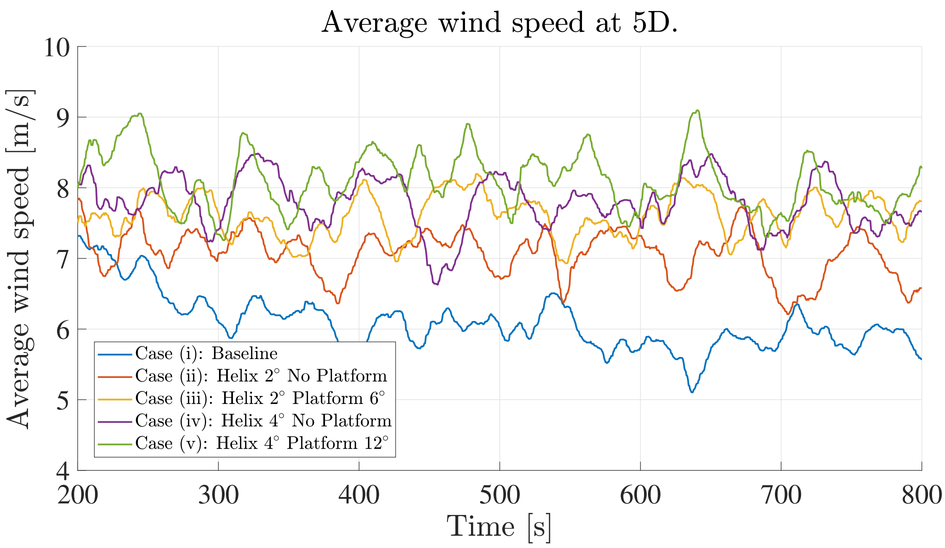


Figure 4: Average velocity across rotor disk for hypothetical turbine located 5D downwind.

2, where the average wind speed and the percentage change with respect to the baseline are presented. The time averaged data confirms the picture sketched by the data presented in Figure 4. Allowing the platform to yaw gives a noticeable increase in average wind velocity at 5D downwind.

Figure 5 shows a top view of the wake for cases (iii), (iv) and (v) for visual comparison between when the Helix is applied on a floating turbine and a bottom-fixed turbine. The helicoidal characteristic shape in the wake can clearly be identified. When the platform is yawing it is less present. This is partly explained by the fact that the wake seems to recover to its free stream velocity at an earlier distance. This is confirmed in Table 2 with the higher

Case:	Case (i)	Case (ii)	Case (iii)	Case (iv)	Case (v)
Average Wind-speed [m/s]	5.95 (-)	7.05 (+18.3%)	7.59 (+27.5%)	7.78 (+30.8%)	8.07 (+35.6%)

Table 2: Summary of the results for the Helix for the average wind speed at 5D distance downwind. Percentages are with respect to case (i).

average velocity at 5D downstream. From the velocity plot it is difficult to determine if the wake is also significantly deflected due to the yawing behaviour, and judging from this snapshot it seems like the wake mixes faster resulting in earlier recovery.

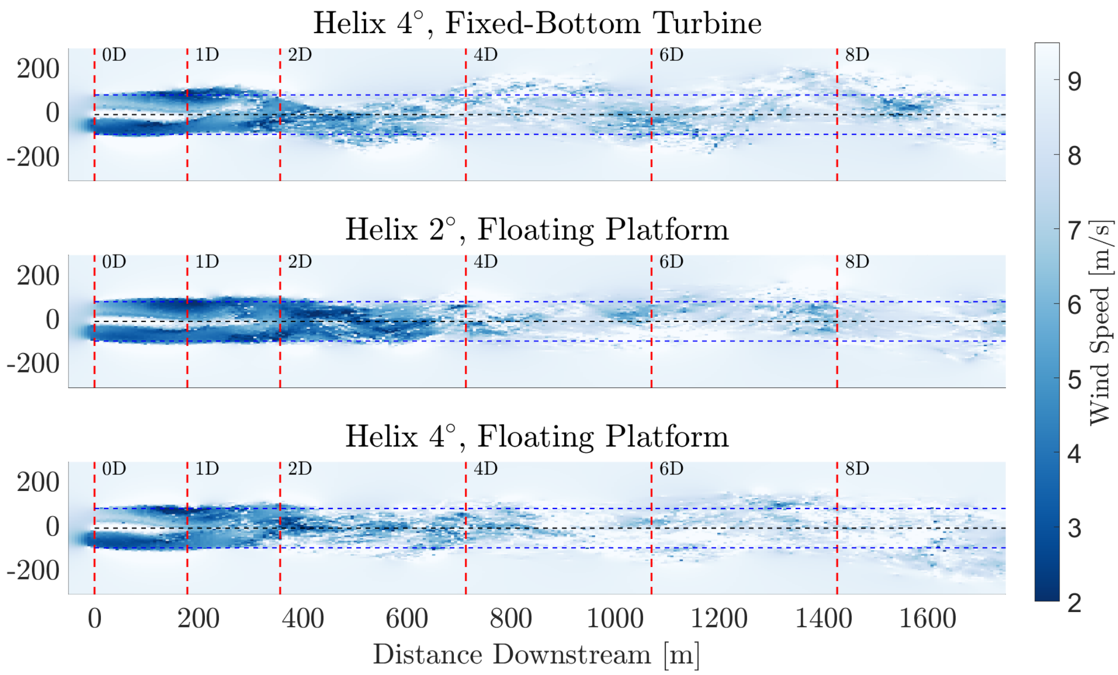


Figure 5: Top down view of the wake for cases (iii), (iv) and (v). The floating turbines are moving with a predefined motion.

These results are also greatly influenced by the fact that the pitching frequency is close to the eigenfrequency of the platform. The large yawing motion, up to ±12°, in combination with the Helix disturbs the wake significantly more than just the Helix. Furthermore, the phase difference at $St = 0.25$ for the yaw motion is close to ideal as seen in Section 2.2. However, being close to the eigenfrequency does introduce uncertainty in the phase offset of the platform movement with respect to the Helix as was discussed in Section 2. A small change in Strouhal number could result in significantly different platform yawing behaviour with respect to the Helix. This is investigated with cases (vi) till (ix), of which only the average wind speed is summarized in Table 3.

As shown in Section 2, when the platform is moving in anti-phase with the yaw pitch, θ_{yaw}, the yaw moment and wake deflection are enhancing one another. This is also the case when the Helix is applied (i.e. the yaw moment is no longer constant) and the platform is moving. Interestingly, none of the floating cases show a decrease in wind speed when compared to the bottom-fixed case, meaning that regardless of phase offset, the platform movement has a positive contribution to the average wind speed behind the turbine.

Case:	Case (ii)	Case (vi)	Case (vii)	Case (viii)	Case (ix)
Average Wind-speed [m/s]	7.05 (-)	7.26 (+3.0%)	7.30 (+3.6%)	7.61 (+7.9%)	7.27 (+3.1%)

Table 3: Summary of the results for the Helix with the platform moving at different phase differences. The average wind speed at 5D distance downwind is shown. Percentages are with respect to case (ii).

5. Discussion and Prospect of Further Research

The results show that allowing platform movement can positively contribute to achieve higher wind speeds downstream. It is already known that the Helix wake-mixing technique achieves this by destabilizing the wake, which causes the re-energizing process to initiate earlier compared to a undisturbed wake. When the platform is yawing, as would be the case with the Helix, the wake is not only being destabilized, but also deflected. Further investigation is required to see if this also further increases the wake-mixing. It could be that purely the deflection of the wake away from the downwind turbine already explains the gain in wind speed.

It is important to note that some simplifications are made to the simulations: (1) the inflow is uniform, (2) there are no waves present and, (3), the movement is restricted to only the yaw motion. In [15] it was shown that when turbulent inflow is considered, the Helix still contributes positively to the downstream wind speed. It is expected that adding turbulence to the inflow will have similar impact on the results of the FOWT. Moreover, when the presence of waves is considered, the motion of the platform will be influenced. These resulting motions could either counteract or amplify the motions that promote wake mixing and/or deflection. Therefore, it is important that any wake-mixing control should also account for the waves hitting the platform. Finally it should be noted that the movement will not be restricted to the yawing motion, but rather the turbine will yaw, roll and pitch at various amplitudes. Based on the results in the presented cases, it is expected that these extra movements to positively contribute to the goal of minimizing wake interaction. The influence these motion(s) have on the wake are dependent on two factors. First, the platform should display significant movement when subjected to a blade pitch input and, second, this movement should also be beneficial for wake mixing. The synergy between wake mixing strategies and the platform dynamics raises an interesting discussion point: As knowledge on wake dynamics advances and the wind energy community better understands how to manipulate the wake, floating platforms could be designed or adjusted to accommodate this new knowledge. This design method, in which controls are an integral part of the overall design, is called co-design. If it is possible to easily excite the desired platform motion it potentially leaves headroom in the control action for the turbine(s) for other tasks.

Finally, it should be investigated if the motion of the platform has a significant impact on the loading of the turbine. Research has already shown that the Helix increases the DEL on the blades and tower of the turbine [25]. The lower blade pitching amplitudes can be considered positive for the turbine loading but this could be negated by the extra platform motion. Another aspect that should be taken into account is that, as the downwind turbine experiences a higher wind speed, the loading on that turbine will also increase. Therefore, it is important to not only look at the first turbine, but the entire wind farm when assessing the loads.

6. Conclusion

In this study the effect of the Helix mixing strategy on a floating offshore wind turbine and its wake is investigated. Two questions that needed to be answered were, first, how much does the FOWT actually move under varying force inputs and, second, what impact will this motion have on the wake looking at from a wake mixing perspective. Using simulations in OpenFast it was found that the specific combination of turbine and platform exhibited significant yaw motion when the Helix was enabled. It was found that this FOWT has an eigenfrequency in yaw motion close to $St = 0.25$. The influence of the platform motion is investigated using Qblade, which shows that when the platform is yawing, the windspeed 5D downstream is significantly increased compared to a fixed-bottom turbine. On average the windspeed is increased by 5 to almost 10% for otherwise identical systems. It can be seen by visual inspection that a combination of the Helix and the platform motion increases the wake recovery compared to the fixed-bottom Helix case. It is important to state that this paper should be considered as a proof-of-concept. In the future the full range of motions of the FOWT should be considered which is expected to have a positive contribution. Furthermore, this work shows that by utilizing the platform dynamics, less control action is required to achieve similar increases in downstream wind speed when compared to a fixed-bottom turbine. This could open up interesting possibilities where FOWTs are designed such that the platform motion accounts for the wake-mixing. Further research is required to better understand the interaction between turbine control, platform design, platform motions and wake dynamics.

7. Acknowledgements
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